OCR
RECOGNISING ACHIEVEMENT

Monday 14 January 2013 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Express $\dfrac{5x}{(x-1)(x^2+4)}$ in partial fractions. **[5]**

2 The equation of a curve is $y = \dfrac{x^2-3}{x-1}$.

(i) Find the equations of the asymptotes of the curve. **[3]**

(ii) Write down the coordinates of the points where the curve cuts the axes. **[1]**

(iii) Show that the curve has no stationary points. **[3]**

(iv) Sketch the curve and the asymptotes. **[3]**

3 By first expressing $\cosh x$ and $\sinh x$ in terms of exponentials, solve the equation

$$3\cosh x - 4\sinh x = 7,$$

giving your answer in an exact logarithmic form. **[6]**

4 You are given that $I_n = \int_0^1 x^n e^{2x}\,dx$ for $n \geqslant 0$.

(i) Show that $I_n = \frac{1}{2}e^2 - \frac{1}{2}nI_{n-1}$ for $n \geqslant 1$. **[4]**

(ii) Find I_3 in terms of e. **[4]**

5 You are given that $f(x) = e^{-x}\sin x$.

(i) Find $f(0)$ and $f'(0)$. **[3]**

(ii) Show that $f''(x) = -2f'(x) - 2f(x)$ and hence, or otherwise, find $f''(0)$. **[4]**

(iii) Find a similar expression for $f'''(x)$ and hence, or otherwise, find $f'''(0)$. **[2]**

(iv) Find the Maclaurin series for $f(x)$ up to and including the term in x^3. **[2]**

6 By first completing the square, find $\int_0^1 \frac{1}{\sqrt{x^2+4x+8}}\,\mathrm{d}x$, giving your answer in an exact logarithmic form. **[6]**

7 A curve has polar equation $r = 5\sin 2\theta$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Sketch the curve, indicating the line of symmetry and stating the polar coordinates of the point P on the curve which is furthest away from the pole. **[4]**

(ii) Calculate the area enclosed by the curve. **[3]**

(iii) Find the cartesian equation of the tangent to the curve at P. **[3]**

(iv) Show that a cartesian equation of the curve is $\left(x^2+y^2\right)^3 = (10xy)^2$. **[3]**

8 It is required to solve the equation $\ln(x-1) - x + 3 = 0$.

You are given that there are two roots, α and β, where $1.1 < \alpha < 1.2$ and $4.1 < \beta < 4.2$.

(i) The root β can be found using the iterative formula

$$x_{n+1} = \ln(x_n - 1) + 3.$$

(a) Using this iterative formula with $x_1 = 4.15$, find β correct to 3 decimal places. Show all your working. **[2]**

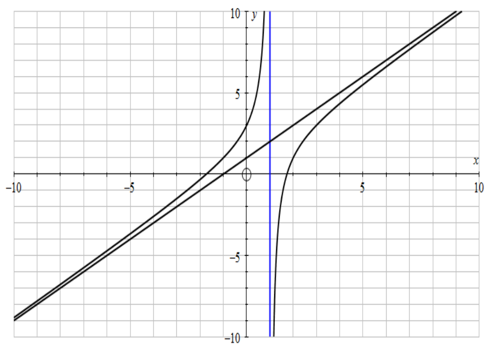
(b) Explain with the aid of a sketch why this iterative formula will not converge to α whatever initial value is taken. **[3]**

(ii) **(a)** Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{n+1} = \frac{3 - 2x_n - (x_n - 1)\ln(x_n - 1)}{2 - x_n}.$$ **[5]**

(b) Use this formula with $x_1 = 1.2$ to find α correct to 3 decimal places. **[3]**

Question			Answer	Marks	Guidance	
1			$\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+4}$ $\Rightarrow 5x \equiv A(x^2+4)+(Bx+C)(x-1) \quad [+D(x-1)(x^2+4)]$	B1	Sight of expression	Allow addition of constant
			Equate coefficients or substitute values for x	M1	For Equating 3 coeffs or sub 3 times	
			$\Rightarrow A=1$ $B=-1$	A1	For one value (not D)	
			$C=4$	A1	For 2nd and 3rd values (not D)	
			$\Rightarrow \dfrac{5x}{(x-1)(x^2+4)}=\dfrac{1}{(x-1)}+\dfrac{4-x}{(x^2+4)}$	A1	For final answer expressed properly	
				[5]		

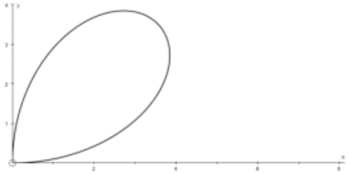
Question			Answer	Marks	Guidance	
2	(i)		$x = 1$ $y = \dfrac{x^2-3}{x-1} = \dfrac{(x-1)(x+1)-2}{x-1} = x+1 \left[-\dfrac{2}{x-1}\right]$ $\Rightarrow y = x+1$	B1 M1 A1 **[3]**	 Or long division with quotient $x +$... Must be stated	
2	(ii)		(0,3) $(\sqrt{3},0)$ and $(-\sqrt{3},0)$	B1 **[1]**	All three	Allow when x = 0, y = 3, etc but do NOT allow $y = 3$, etc
2	(iii)		$\dfrac{dy}{dx} = \dfrac{2x(x-1)-(x^2-3)}{(x-1)^2} = \dfrac{x^2-2x+3}{(x-1)^2}$ $= \dfrac{(x-1)^2+2}{(x-1)^2} > 0$ for all x. So no turning points.	M1 A1 A1 **[3]**	Differentiate Gradient function Conclusion	Alternative method: Diffn final expression from (i) $\dfrac{dy}{dx} = 1 + \dfrac{2}{(x-1)^2}$ >1 so no turning points. Or "b^2 – 4ac"=-8 < 0 so no roots.
2	(iv)			B1 B1 B1 **[3]**	Correct shape going through axes at correct points which must be stated. Correct asymptotes included Approaches correct asymptotes correctly	Allow omission of (0, 3) if not in (ii). Oblique asymptote can be $y = x + c$ with $c \neq 1$

Question			Answer	Marks	Guidance	
3			$3\dfrac{e^x+e^{-x}}{2}-4\dfrac{e^x-e^{-x}}{2}=7$	M1	Use of formulae	
			$\Rightarrow 3\left(e^x+e^{-x}\right)-4\left(e^x-e^{-x}\right)=14$	A1	Correct equation	
			$\Rightarrow -e^x+7e^{-x}=14$ $\Rightarrow e^{2x}+14e^x-7=0$	A1	Correct quadratic equation in e^x	
			$\Rightarrow e^x=\dfrac{-14\pm\sqrt{196+28}}{2}$	M1	Solve quadratic	
			$\left[e^x>0\right]$ so $e^x=\dfrac{-14+\sqrt{196+28}}{2}$ $=-7+\sqrt{56}$	A1	Correct value for e^x (ignore -ve value)	
			$\Rightarrow x=\ln\left(2\sqrt{14}-7\right)$	A1	One value only with statement of rejection of invalid value for e^x	
				[6]		
			Alternative Make sinh or cosh the subject, square, use $c^2 - s^2 = 1$	M1 A1		
			Gives $7s^2 + 56s + 40 = 0$ Or $7c^2 + 42c - 65 = 0$	A1		

Question			Answer	Marks	Guidance	
4	(i)		$I_n = \int_0^1 x^n . e^{2x}\, dx.$			
			Set $u = x^n \quad du = nx^{n-1} dx$	M1	Integration by parts	
			$dv = e^{2x}\, dx \quad v = \frac{1}{2}e^{2x}$	A1	Correct way round and correct diffn	
			$\Rightarrow I_n = \int_0^1 x^n e^{2x}\, dx = \left[\frac{1}{2}x^n e^{2x}\right]_0^1 - \frac{1}{2}n\int_0^1 x^{n-1}e^{2x}\, dx$	A1	Indefinite form acceptable	
			$I_n = \frac{1}{2}e^2 - \frac{1}{2}nI_{n-1}$	A1	Using limits	
				[4]		
4	(ii)		$I_0 = \int_0^1 e^{2x}\, dx = \frac{1}{2}\left[e^{2x}\right]_0^1 = \frac{1}{2}\left(e^2 - 1\right)$	M1 A1	Attempt to find I_0 or I_1.	
			$I_1 = \frac{1}{2}e^2 - \frac{1}{2}I_0 = \frac{1}{2}e^2 - \frac{1}{2}\left(\frac{1}{2}\left(e^2 - 1\right)\right) = \frac{1}{4}e^2 + \frac{1}{4}$	M1	Using this to progress, dep	
			$I_2 = \frac{1}{2}e^2 - I_1 = \frac{1}{2}e^2 - \left(\frac{1}{4}e^2 + \frac{1}{4}\right) = \frac{1}{4}e^2 - \frac{1}{4}$			
			$I_3 = \frac{1}{2}e^2 - \frac{3}{2}I_2 = \frac{1}{2}e^2 - \frac{3}{2}\left(\frac{1}{4}e^2 - \frac{1}{4}\right) = \frac{1}{8}e^2 + \frac{3}{8}$	A1		
				[4]		

Question			Answer	Marks	Guidance	
5	(i)		$f'(x) = -\sin x.e^{-x} + \cos x.e^{-x}$ $\Rightarrow f'(0) = 1$ $f(0) = 0$	M1 A1 A1 **[3]**	Diffn using product correctly. For **both** values www	
5	(ii)		$f'(x) = \cos x.e^{-x} - \sin x.e^{-x} = \cos x.e^{x} - f(x)$ $f''(x) = -f'(x) - \cos x.e^{-x} - f(x)$ $= -f'(x) - f'(x) - f(x) - f(x)$ $f''(x) = -2f'(x) - 2f(x)$ OR $-2\cos x.e^{-x}$ Showing the two equal $f''(0) = -2$	M1 A1 A1 A1 **[4]**	Diffn	
5	(iii)		$f''(x) = -2f'(x) - 2f(x)$ $\Rightarrow f'''(x) = -2f''(x) - 2f'(x)$ **oe** $\Rightarrow f'''(0) = 4 - 2 = 2$	B1 B1 **[2]**	Not involving trig or exp fns	=-f''+2f Or 2f' + 4f
5	(iv)		$f(x) = x - x^2 + \dfrac{x^3}{3}$	M1 A1 **[2]**		
			Alternative: Write down correct series expansion for e^{-x} and sin$x$ and multiply	M1 A1		

Question			Answer	Marks	Guidance	
6			$x^2+4x+8=(x+2)^2+4$	M1 A1	Complete the square in order to use standard form	
			$\int_0^1 \frac{1}{\sqrt{x^2+4x+8}}\,dx=\int_0^1 \frac{1}{\sqrt{(x+2)^2+4}}\,dx$	M1	Use correct standard form in integration	
			$=\left[\sinh^{-1}\frac{x+2}{2}\right]_0^1=\sinh^{-1}\left(\frac{3}{2}\right)-\sinh^{-1}1$	A1	Answer in $\sinh^{-1}$ form	
			$=\ln\left(\frac{3}{2}+\sqrt{1+\frac{9}{4}}\right)-\ln\left(1+\sqrt{2}\right)=\ln\left(\frac{3}{2}+\sqrt{\frac{13}{4}}\right)-\ln\left(1+\sqrt{2}\right)$	M1	Attempt to turn into log form	
			$=\ln\left(\frac{3+\sqrt{13}}{2+2\sqrt{2}}\right)$	A1	www isw	
				[6]		
			Alternative for last 4 marks			
			$\int_0^1 \frac{1}{\sqrt{(x+2)^2+4}}\,dx=\left[\ln\left((x+2)+\sqrt{(x+2)^2+4}\right)\right]_0^1$	M1 A1 M1	Attempt to use Standard form Limits	
			$=\ln\left(3+\sqrt{13}\right)-\ln\left(2+\sqrt{8}\right)=\ln\left(\frac{3+\sqrt{13}}{2+2\sqrt{2}}\right)$	A1	www isw	
			Alternative for last 4 marks			
			$x+2=2\tan\theta\Rightarrow I=\left[\ln\left(\sec\theta+\tan\theta\right)\right]_{\pi/4}^{\tan^{-1}3/2}$	M1 A1	Substitution Indefinite integral	
			$=\ln\left(\frac{3}{2}+\frac{\sqrt{13}}{2}\right)-\ln\left(1+\sqrt{2}\right)=\ln\left(\frac{3+\sqrt{13}}{2+2\sqrt{2}}\right)$	M1 A1	Deal with limits www isw	

Question			Answer	Marks	Guidance	
7	(i)		 P is at $r = 5,\ \theta = \frac{\pi}{4}$	B1 B1 B1 B1 **[4]**	Enclosed loop with axes tangential $\theta = \frac{\pi}{4}$ is a line of symmetry drawn and named For both	Ignore anything in other quadrants
7	(ii)		Area $= \frac{1}{2}\int_0^{\pi/2} r^2\ d\theta = \frac{1}{2}\int_0^{\pi/2} 25\sin^2 2\theta\ d\theta$ $= \frac{25}{4}\int_0^{\pi/2}(1-\cos 4\theta)\ d\theta = \frac{25}{4}\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\pi/2}$ $= \frac{25}{4}\left(\left(\frac{\pi}{2}-0\right)-(0)\right) = \frac{25\pi}{8}$	M1 M1 A1 **[3]**	Correct formula with r substituted. Correct method of integration including limits www	
7	(iii)		Equation is of the form $x + y = c$ P is $\left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$ **oe** $\Rightarrow x + y = 5\sqrt{2}$	B1 B1 B1 **[3]**	 Ft. $x + y = c$ where c comes from their P.	
7	(iv)		$r = 5\sin 2\theta = 10\sin\theta\cos\theta$ $\Rightarrow r^2 = 100\sin^2\theta\cos^2\theta = 100\left(\frac{y}{r}\right)^2\left(\frac{x}{r}\right)^2$ $\Rightarrow \left(x^2+y^2\right)^3 = 100x^2y^2$	M1 M1 A1 **[3]**	Square and convert r^2 Substitute for r and θ ***NB Answer given***	

Question			Answer	Marks	Guidance	
8	(i)	(a)	$x_1 = 4.15,\quad x_2 = 4.1474....$ $x_3 = 4.1465....,\quad x_4 = 4.1463....$ $\beta = 4.146$	M1 A1 **[2]**	Using iterative formula at least once using at least 4dp www	All iterates must be seen
8	(i)	(b)	Staircase diagram will always move to upper root	B1 B1 B1 **[3]**	Sketch showing an example $x_1 > \alpha$ Example with $x_1 < \alpha$ Statement Dep on 1st two B	Ignore any statement when $x_1 > \beta$
8	(ii)	(a)	$\ln(x-1) = x-3 \Rightarrow \ln(x-1)-(x-3)=0$ $\Rightarrow \mathrm{f}(x) = \ln(x-1)-(x-3)$ $\Rightarrow \mathrm{f}'(x) = \dfrac{1}{x-1} - 1$ $\Rightarrow x_{n+1} = x_n - \dfrac{\ln(x_n-1)-(x_n-3)}{\dfrac{1}{x_n-1}-1}$ $= x_n - \dfrac{(x_n-1)(\ln(x_n-1)-(x_n-3))}{1-(x_n-1)}$ $= \dfrac{x_n(2-x_n)+(x_n-1)(x_n-3)-(x_n-1)\ln(x_n-1)}{2-x_n}$ $= \dfrac{2x_n - x_n^2 + x_n^2 - 4x_n + 3 - (x_n-1)\ln(x_n-1)}{2-x_n}$ $\Rightarrow x_{n+1} = \dfrac{3-2x_n-(x_n-1)(\ln(x_n-1)}{2-x_n}$	M1 M1 M1 A1 A1 **[5]**	Get equation in correct form Differentiate Use correct formula Mult by $(x-1)$ **soi**	

Question			Answer	Marks	Guidance	
8	(ii)	(b)	1.2 1.152(359) 1.152359 1.158448 1.158448 1.158594 Root = 1.159 1.158594 1.158594	B1 B1 B1 **[3]**	For x_2 For enough iterates to determine 3dp www	Allow 3 dp x_2 must be right for last B1. Any error is likely to be self-correcting